Signed Numbers – negatives and all those minus signs

Mathematicians use and reuse the same symbols sometimes, changing the meaning by where the symbol hangs on a number. For example:

 means three unit steps to the left of zero while  means just a little bit less than positive three (numbers like 2.91 and 2.999). Further  . And  is the second three is a list of threes. While .

We’ll focus on having the negative sign out front of the number in this section.

First let’s back all the way up to multiplication.

 in a very basic way says “take 2 of those 3’s and add them up”. It’s actually a kind of instruction. Multiplication by −1 is also an instruction. It means “go to the other side of zero”. So if you take 5 and multiply by −1 you end up on the left side of zero at −5. In fact, if we write out −5 the long way we get −5 = −1(5). We don’t want to keep writing it the long way, though, so we contract it. Just like nobody actually says “cannot”; everybody says “can’t”.

Now have you ever wondered why “a minus times a minus is a plus”? Here’s why.



The first minus one says go to the other side of zero. The second one says go to the other side of zero…you go negative and then go BACK to the positive side when there are two multiplied negative ones.

Some practice:



This gets conflated into subtraction and makes the confusion even greater.

 This is pretty clear. Subtraction is a form of adding with negative numbers, BUT let’s look at a more difficult one:



Notice that I wrote this one out the long way, too. As a form of addition with that “go to the other side of zero and go back” instruction clearly showing.

It’ll take some thinking to go to the longer way of writing it out and some work, but pretty soon you’ll be saying “minus times a minus is a plus” and you’ll know exactly what you are talking about!



You can change the problem to an addition with a negative number and it will commute around the addition symbol. But use care.  

Practice:



Fill in the blank with < or >



Now there is a wrinkle we’ll bring up again and again. When you have a negative number on one side of an inequality symbol clearing the negative REVERSES the inequality. Let’s look at why:

 now you want positive x compared to a number.

When you multiply the sides of the inequality, you are sending the ray to the other side of zero, so if you switch from a negative to a positive, you are taking a greater than and by smoothly flipping it to the other side of zero, the symbol flips, too. Try this with your hands…your thumb switches sides. So with the symbol.



Try some values above the answer – i.e. you forgot to switch the symbol to see that this works well.

Some problems



Try numbers in your answer zone and simulate forgetting to switch by trying numbers on the other side of your answer zone.